[4] Melnik, R. E., "A conical thin-shock-layer theory uniformly valid in the entropy layer," Flight Dynamics Lab. TDR 64-82 (January 1965).

[5] Scheuing, R. A., Mead, H. R., Brook, J. W., Melnik, R. E., Hayes, W. D., Gray, K. E., Donaldson, C. duP., and Sullivan, R. D., "Theoretical prediction of pressures in hypersonic flow with special reference to configurations having attached leading edge shocks," Aeronautical Systems Div. ASD TR 61-60, part I (May 1962).

[6] Tsien, H. S., "The Poincaré-Lighthill-Kuo method," *Advances in Applied Mechanics* (Academic Press, New York, 1956), Vol. 4.

Radiation Stresses on Real Surfaces

D. K. Edwards* and J. T. Bevans†
TRW Space Technology Laboratories, Redondo Beach, Calif.

Nomenclature

- F = arbitrary function of θ and φ, Eq. (4)
- G = solar irradiation normal to a surface, ergs/sec-cm²
- I = spectral radiant intensity, ergs/sec-cm²-sr-μ; I^+ = leaving, I^- = incident, I_b = ideal Planckian radiator
- P = surface stress, dynes/cm²-sr-μ; P_x, P_y, and P_z = stresses in the x, y, and z directions of the surface
- T = absolute temperature, °K
- c = velocity of light, 2.998 × 10¹⁰ cm/sec
- = reflection distribution function, as defined by $I^+(\theta_2, \varphi_2) = f(\theta_1, \varphi_1, \theta_2, \varphi_2)I^-(\theta_1, \varphi_1) \cos\theta_1 d\Omega_1$ where Ω_1 is the solid angle of the source
- h = Planck's const, 6.625 × 10⁻²⁷ erg-sec
- ϵ = emittance
- θ = polar angle of spherical coordinate system; θ_1 = incident, θ_2 = leaving, θ_0 = incidence for solar irradiation
- λ = wavelength, μ; used as a subscript to denote the monochromatic dependency of the subscripted quantity
- ν = frequency, sec⁻¹
- π = 3.14159
- ρ = reflectance
- φ = azimuthal angle of spherical coordinate system; φ_1 = incident, φ_2 = leaving, φ_0 = incidence for solar irradiation

RECENTLY there have been a number of papers on radiation forces. Clancy and Mitchell[1] have shown the effect of radiation-induced torques upon satellite attitude under the assumption of specularly reflecting surfaces. A similar analysis was performed by Polyakhova with the specular assumption. Holl[3] utilized Fresnel's equations as a more realistic approach to the angular variation of reflectance. The purpose of this note is to give the general expression for the radiation stress on a surface of arbitrary reflecting properties, i.e., real, not necessarily perfectly specular or perfectly diffuse. Knowledge of these stresses is important in accurately predicting disturbance torques on gravity-gradient stabilized systems.

In the computation of radiation transfer between real surfaces, spectral radiant intensity $I(\theta, \varphi, \lambda)$ (ergs/sec-cm²-sr-μ) is employed to give the power in direction θ, φ per unit normal area, solid angle, and wavelength bandwidth. The quantity I can be determined when the reflection distribution function is known. These quantities, the radiant intensity and reflection distribution function, can be employed to calculate force per unit area on real surfaces caused by impinging or departing streams of photons. Such surface stresses (not just pressures) are needed in order to calculate forces and moments causing space vehicles to change trajectory and attitude.

According to Einstein's photoelectric law, any particle carries energy $h\nu$. A photon traveling at the speed of light carries momentum $h\nu/c$. The momentum stream per unit normal area, solid angle, and wavelength bandwidth in the direction θ, φ is then I/c (dynes/cm²-sr-μ). This stream gives rise to a force according to Newton's laws. The stress vector on a surface is thus equal to minus the integral of the momentum stream over all solid angles:

$$\begin{vmatrix} P_x \\ P_y \\ P_z \end{vmatrix} = -\int_0^\infty \int_0^{2\pi} \int_0^{\pi/2} \begin{vmatrix} \sin\theta \cos\varphi \\ \sin\theta \sin\varphi \\ \cos\theta \end{vmatrix} \left(\frac{I}{c}\right) \sin\theta \cos\theta d\theta \varphi d\lambda \tag{1}$$

Two radiant fluxes must be distinguished, one impinging with (Fig. 1) intensity I^- and the other departing with intensity I^+. If the incident flux I^- is known, the departing flux from a surface in a state of thermodynamic equilibrium at temperature T is given by the reflection distribution function[4] $f(\theta_1, \varphi_1, \theta_2, \varphi_2, \lambda)$ and the directional emissivity (related to an integral of the reflection distribution function).[5] The subscript 1 denotes incidence directions, and subscript 2 denotes departing directions:

$$I^+(\theta_2, \varphi_2, \lambda) = \epsilon(\theta_2, \varphi_2)I_b(T, \lambda) + \int_0^{2\pi} \int_0^{\pi/2} I^-(\theta_1, \varphi_1, \lambda)f(\theta_1, \theta_1, \theta_2, \varphi_2, \lambda) \sin\theta_1 \cos\theta_1 d\theta_1 d\varphi_1 \tag{2}$$

where I_b is the blackbody intensity by Planck's law. Thus the complete expression for the surface stress is

$$\begin{vmatrix} P_x \\ P_y \\ P_z \end{vmatrix} = -\int_0^\infty \int_0^{2\pi} \int_0^{\pi/2} \begin{vmatrix} \sin\theta_1 \cos\varphi_1 \\ \sin\theta_1 \sin\varphi_1 \\ \cos\theta_1 \end{vmatrix} \times \left(\frac{I^-}{c}\right) \sin\theta_1 \cos\theta_1 d\theta_1 d\varphi_1 d\lambda - \int_0^\infty \int_0^{2\pi} \int_0^{\pi/2} \begin{vmatrix} \sin\theta_2 \cos\varphi_2 \\ \sin\theta_2 \sin\varphi_2 \\ \cos\theta_2 \end{vmatrix} \times \left\{\epsilon(\theta_2, \varphi_2, \lambda)\left[I_b\left(\frac{T\lambda}{c}\right)\right] + \int_0^{2\pi} \int_0^{\pi/2} \left(\frac{I^-}{c}\right) \times f(\theta_1, \varphi_1, \theta_2, \varphi_2, \lambda) \sin\theta_1 \cos\theta_1 d\theta_1 d\varphi_1\right\} \sin\theta_2 \cos\theta_2 d\theta_2 d\varphi_2 d\lambda \tag{3}$$

Several special cases of Eq. (3) can be readily integrated to closed form. First, if I^- is nearly collimated as is solar radiation at the earth's distance from the sun, the integration over θ_1 and φ_1 reduces as follows:

$$\int_0^\infty \int_0^{2\pi} \int_0^{\pi/2} I^- \sin\theta_1 \cos\theta_1 F(\theta_1,\varphi_1) d\theta_1 d\varphi_1 d\lambda = G \cos\theta_0 F(\theta_0, \varphi_0) \tag{4}$$

Angle θ_0 is the polar angle between the surface normal and the line of sight to the center of the sun or other source, and φ_0 is the azimuthal angle measured in the plane of the surface. Taking F to be successively each of the functions in Eq. (3) transforms Eq. (3) to

$$\begin{vmatrix} P_x \\ P_y \\ P_z \end{vmatrix} = -\begin{vmatrix} \sin\theta \cos\varphi_0 \\ \sin\theta_0 \sin\varphi_0 \\ \cos\theta_0 \end{vmatrix} \left(\frac{G}{c}\right) \cos\theta_0 - \int_0^\infty \int_0^{2\pi} \int_0^{\pi/2} \left\{\begin{vmatrix} \sin\theta_2 \cos\varphi_3 \\ \sin\theta_2 \cos\varphi_2 \\ \cos\theta_2 \end{vmatrix} \epsilon(\theta_2, \varphi_2, \lambda)\left[I_b\left(\frac{T\lambda}{c}\right)\right] + \left(\frac{G_\lambda}{c}\right) \cos\theta_0 f(\theta_0, \varphi_0, \theta_2, \varphi_2, \lambda)\right\} \sin\theta_2 \cos\theta_2 d\theta_2 d\varphi_2 d\lambda \tag{5}$$

If the reflection distribution function is that for a perfectly diffuse or a perfectly specular surface, then Eq. (5) can be reduced further. For a perfectly diffuse surface, $f(\theta_0, \varphi_0, \theta_0, \varphi_2, \lambda)$

Received August 10, 1964.

* Consultant; also Associate Professor of Engineering, University of California, Los Angeles, Calif. Member AIAA.

† Head, Thermal Radiation Laboratory. Member AIAA.

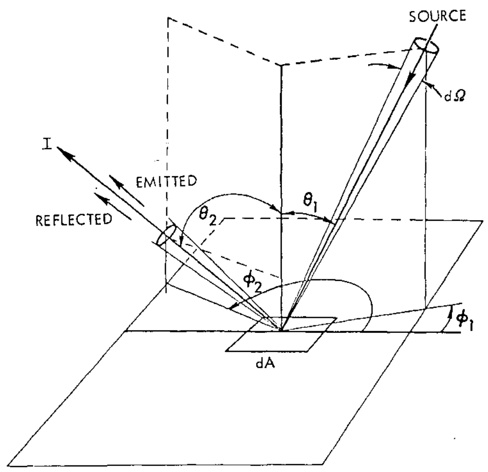

Fig. 1 Geometry of radiation incident and departing from a differential area of a surface.

$= \rho/\pi$ (where ρ is the reflectance of the surface), and Eq. (5) becomes

$$\begin{vmatrix} P_x \\ P_y \\ P_z \end{vmatrix} = - \begin{vmatrix} \sin\theta_0 \cos\varphi_0 \\ \sin\theta_0 \sin\varphi_0 \\ \cos\theta_0 \end{vmatrix} \left(\frac{G}{c}\right) \cos\theta_0 - \int_0^\infty \begin{vmatrix} 0 \\ 0 \\ \frac{2}{3} \end{vmatrix} \times \left(\frac{\rho_\lambda G_\lambda}{c}\right) \cos\theta_0 d\lambda - \int_0^\infty \begin{vmatrix} 0 \\ 0 \\ \frac{2}{3} \end{vmatrix} \epsilon(\lambda)\left[\pi I_B\left(\frac{T\lambda}{c}\right)\right] d\lambda \quad (6)$$

For a perfectly specular sample, the quantity f in Eq. (2) is a delta function so that

$$I^+(\theta_2, \varphi_2, \lambda) = \epsilon(\theta_2, \varphi_2, \lambda) I_b + \rho(\theta_1, \varphi_1, \lambda) I^- \qquad \theta_2 = \theta_1 \qquad \varphi_2 = \varphi_1 + \pi \quad (7)$$

In this case the stress Eq. (5), with $\rho = \rho(\theta_0, \varphi_0)$ averaged over the spectrum corresponding to G, becomes

$$\begin{vmatrix} P_x \\ P_y \\ P_z \end{vmatrix} = - \begin{vmatrix} \sin\theta_0 \cos\varphi_0 (1 - \rho) \\ \sin\theta_0 \sin\varphi_0 (1 - \rho) \\ \cos\theta_0 (1 + \rho) \end{vmatrix} \left(\frac{G}{c}\right) \cos\theta_0 - \int_0^\infty \int_0^{2\pi} \int_0^{\pi/2} \begin{vmatrix} \sin\theta_2 \cos\varphi_2 \\ \sin\theta_2 \sin\varphi_2 \\ \cos\theta_2 \end{vmatrix} \epsilon(\theta_2, \varphi_2, \lambda) \times \left[I_b\left(\frac{T\lambda}{c}\right)\right] \sin\theta_2 \cos\theta_2 d\theta_2 d\varphi_2 d\lambda \quad (8)$$

Equation (3) gives the general case, whereas Eqs. (6) and (8) are special cases for directional irradiation and diffuse and specular reflection, respectively. A final observation that may be made is that, if a maximum solar radiation moment is desired at a grazing angle of incidence, $\theta_0 \simeq 80°–90°$, as on a solar rudder sail, a diffuse reflecting surface is more desirable than a specular surface, and the best surface would be a grating blazed such that I^+ was directed along $\theta_2 = 0$. Such a grating could be made by embossing thin foil, for example.

References

[1] Clancy, T. F. and Mitchell, T. P., "Effect of radiation forces on the attitude of an artificial earth satellite," AIAA J. **2,** 517 (1964).

[2] Polyakhova, Y. N., "Solar radiation pressure and the motion of earth satellites," AIAA J. **1,** 2893 (1963).

[3] Holl, H. B., "The effect of radiation force on satellites of convex shapes," NASA TN D-604 (May 1961).

[4] McNicholas, H. J., "Absolute methods of reflectometry," J. Res. Natl. Bur. Std. **1,** 29 (1928).

[5] Edwards, D. K. and Bevans, J. T., "Thermal radiation characteristics of imperfectly diffuse samples," TRW Space Technology Labs. Rept. 9990-6343-TU-000.

An Experimental Investigation of the Erosive Burning Characteristics of a Nonhomogeneous Solid Propellant

M. J. Zucrow,* J. R. Osborn,† and J. M. Murphy‡
Purdue University, Lafayette, Ind.

Nomenclature

A_{th} = area of the throat of the exhaust nozzle
A_{ts} = port area of the test section
g = acceleration due to gravity
k = ratio of specific heats of the combustion gases
M_{ts} = average Mach number of the combustion gases in the test section
O/F = ratio of the oxidizer to the binder by % wt
p_c = combustion pressure, psia
r_0 = linear burning rate
r = total burning rate, in./sec
r_e = erosive burning rate $(r - r_0)$
R = gas constant
SF = scale factor defined by Eq. (1)
t_i = propellant temperature; i.e., its temperature prior to ignition
t_f = flame temperature of the combustion gases
u_g = average velocity of the combustion gases, fps
u_{tv} = threshold velocity of the combustion gases; velocity above which $r > r_0$
Δt_b = burning time
Δx = change in the position of the burning surface during Δt_b

I. Introduction

In general, the burning rate of a solid propellant, denoted by r, is a complex function of many variables and may be represented by the following functional relation[1]:

$$r = r(p_c, t_f, t_i, u_g, O/F)$$

In addition, it is a function of the composition of the propellant, the oxidizer particle size (in a composite propellant), and the geometry of the propellant grain.

II. Experimental Investigation

The experiments were conducted with two-dimensional rocket motors comprising a gas generator and a test section; the propellant sample was burned in a two-dimensional test section.

Figure 1 illustrates the essential features of the test section; the propellant samples were bonded to its diverging sides. The divergence of the latter was such that the axial velocity of the combustion gases (discharged from the gas generator) was substantially constant in the region where they wetted the propellant samples.

In each experiment, the propellant sample burned in the test section was taken from the same bath of propellant that was burned in the gas generator. Consequently, there was no significant difference in the composition of the gases pro-

Presented as Preprint 64-107 at the AIAA Solid Propellant Rocket Conference, Palo Alto, Calif., January 29–31, 1964; revision received November 2, 1964. The research was sponsored by the Air Force Systems Command, Edwards Air Force Base, Calif., under Contract AF 04(611)-7445. Reproduction in whole or part is permitted for any purpose of the United States government. The authors express their thanks to the following individuals who were helpful in either conducting the research or preparing the paper: R. J. Burick, S. Lowe, W. Bloyd, and H. M. Cassiday.

* Atkins Professor of Engineering. Fellow Member AIAA.

† Professor of Mechanical Engineering. Associate Fellow Member AIAA.

‡ Research Assistant, Jet Propulsion Center; now Senior Engineer, Thiokol Chemical Corporation, Huntsville, Ala. Member AIAA.